The probability that a pair \( p_a \) of people from a set of \( n \) people have the same birthday is the same as the probability \( p_b \) of another pair of people who have the same birthday:

\[
p_a = p_b = \frac{n! \cdot \left(\frac{365}{n}\right)}{365^n}
\]

The probability that the two pairs \( a \) and \( b \) have a \textit{common} birthday is \( p_c = p_a \cdot p_b \):

\[
p_c = p_a \cdot p_b
\]

\[
= \left(\frac{n! \cdot \left(\frac{365}{n}\right)}{365^n}\right)^2
= \frac{n! \cdot \left(\frac{365}{n}\right) \cdot n! \cdot \left(\frac{365}{n}\right)}{365^n \cdot 365^n}
= \frac{n! \cdot \frac{365!}{n!(365-n)!} \cdot n! \cdot \frac{365!}{365^n}}{365^n}
= \frac{n! \cdot \frac{365!}{n!(365-n)!} \cdot n! \cdot \frac{365!}{n!(365-n)!}}{365^n \cdot 365^n}
= \frac{365!}{(365-n)! \cdot 365^n} \cdot \frac{365!}{(365-n)! \cdot 365^n}
= \frac{365!^2}{(365-n)!^2 \cdot 365^{2n}}
\]